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# DERIVATIVES IN PROBABILISTIC SIMULATIONS OF ELECTRIC POWER SYSTEM OPERATIONS

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Abstract - Derivatives associated with production simulations provide valuable guidance in electric system planning, but they have been calculated only recently. We present a closed-form and computationally efficient expression for the derivatives, based on the cumulant implementation of the equivalent load methodology. The paper calculates the derivative of energy served by each generating resource with respect to the capacity of any resource. The general method can be extended to calculate other derivatives of interest for resource planning.

## I. INTRODUCTION

Derivatives provide valuable information when minimizing the value of an objective function. For each set of decisions, the derivatives of the objective function with respect to the decision options show a way to change the decisions that will reduce the value of the objective function. When the best set of decisions is arrived at, the derivatives indicate that no further movement is desirable. Derivatives also indicate the marginal value of resources [1].

Electric power planning is full of situations in which an objective function is to be minimized. For example, utilities develop capacity expansion plans to minimize the cost of providing service to the utility's customers, or operate generating resources to minimize the cost of serving a load with a given set of plants.

Until recently, the use of derivatives has been noticeably absent in the methods used in electric power planning. Some power planning methods [2] use enlightened search to find the best decisions. Others use linear programming [3], where the simplicity of the problem structure allows solutions to be obtained without conscious examination of the derivatives.

The calculation of derivatives has been presented elsewhere [4,5,6], and calculation has been implemented in two utility planning computer codes. This paper explicitly describes the calculation procedure and presents results.

The absence of the use of derivatives was not without reason. Calculating many of the potentially helpful derivatives would require calculating the effect of changing a decision on the operating cost of an electric power system. Analysts usually estimate this operating cost with a production simulation, a complex mapping from a time-varying load (demand) and a set of randomly available resources into resource operation and associated costs. Only recently have methods for taking derivatives associated with this complex mapping been determined.

It is possible to obtain derivatives numerically. The production simulation is run with a base set of decision settings. One decision setting is changed by a small amount and the production simulation is re-run. The change in an output divided by the change in the decision setting provides an estimate of one of the derivatives. However, this procedure is usually too expensive to use in practice. A production simulation must be run for each desired derivative, a burdensome requirement for the dimensionality of the decision sets found in electric power planning.

This paper describes an analytical method for evaluating derivatives. The method builds upon recent advances in production simulation. We first describe production simulation to introduce our necessarily complex notation. Next, we describe the recent advances that make our method practical. We then develop the derivatives, a straightforward if exacting application of the differential calculus. Finally, we provide an example to verify our results and illustrate how they can be used in system planning.

## II. PRODUCTION SIMULATION

Production simulation takes as given a set of randomly available resources and a time-varying load. The simulation determines how much energy is served with each resource. A resource is any method that can serve load. Conventional electric generating units such as coal-fired plants are resources. Less obvious examples of resources are load management technologies such as devices for terminating electric service to air conditioners, or emergency actions such as appeals to industrial customers to reduce electric demand. These resources experience random periods when they are unavailable to serve load; the inability of a plant to serve load is called an outage. The load is the time series demand for electric power placed on the electric system by the customers.

Finding a computationally efficient method for explicitly representing the interaction between the randomly available resources and the time-varying load proved difficult until the concept of equivalent load was put forth by Baleriaux, Jamoulle, and Guertechin in 1967 [7,8]. Load variation over time is represented probabilistically as the load duration curve, ldc,* in Figure 1. The figure shows a complementary cumulative probability distribution on load; that is, the graph shows the probability (percent of time) that the load is greater than or equal to the load shown on the horizontal axis.

Let D be the demand or load; it varies across time. We represent this phenomenon with a probability density function d(D), the density function per unit time on demand D. The load duration curve, ldc(D), is defined as

$$\mathrm{ldc}(D) = 1 - \int_{D \geq X} d(X)$$

*Our notational conventions are explained in Appendix A.

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This is the curve shown in Figure 1.

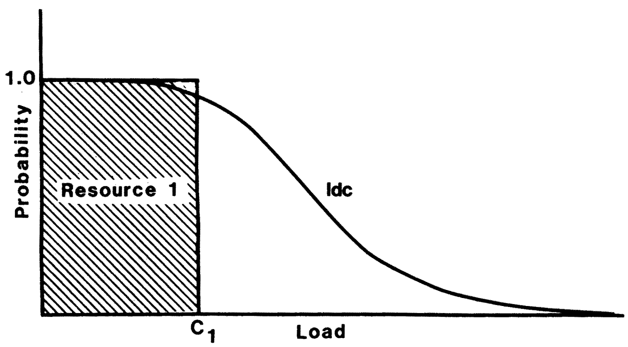


Fig. 1. Dispatching the First Resource

The production simulation starts with the load duration curve. Each resource can serve load any time it is available if load is present. To minimize the operating cost, the resources are dispatched in order of increasing operating cost, the loading order. The least cost resource is dispatched as often as possible before dispatching the next least costly resource.

The load served by the first resource is easy to calculate. Figure 1 shows the first resource's capacity superimposed on the load duration curve. If the resource generated at full capacity all of the time, the energy generated by the resource would be the shaded area under the load duration curve inside the bar representing the unit. If each unit of capacity of the resource can produce energy with probability p, then the energy produced by the resource is p times the shaded area.

The second resource faces the original load minus the load served by the first resource. Let $C_1$ be the total capacity of the first resource and $O_1$ its outage level. Then the second resource faces demand equal to

$$D - (C_1 - O_1) = D + O_1 - C_1$$

We define $D + O_1$ to be the equivalent load after the first resource is dispatched; this equivalent load is denoted by $EL_1$.

The demand and the resource outage state are random variables. Let the probability density of the outage states for the first resource be $o_1(O_1)$. The equivalent load is equal to $EL_1$ when demand is equal to D and the outage state is equal to $EL_1 - D$. The probability density function for the equivalent load after one resource has been dispatched is

$$el_1(EL_1) = \int_D d(D)\, o_1(EL_1 - D)$$

if the load and the outage state are independent random variables. The process in the above equation is called convolution of the two random variables.

An equivalent load (duration) curve ($elc_i$) is related to the probability density on equivalent load ($EL_i$) in the same manner in which the load duration curve is related to the density function on demand. That is, the equivalent load curve is the complementary cumulative probability distribution function of the equivalent load

$$elc_1(EL_1) = 1 - \int_{EL_1 \geq X} el_1(X)$$

The equivalent load curve is subscripted by the number of resources that have been dispatched; there is a different equivalent load curve for each resource. Figure 2 shows the first equivalent load curve. It lies to the right of the load duration curve if the probability of an outage is greater than zero.

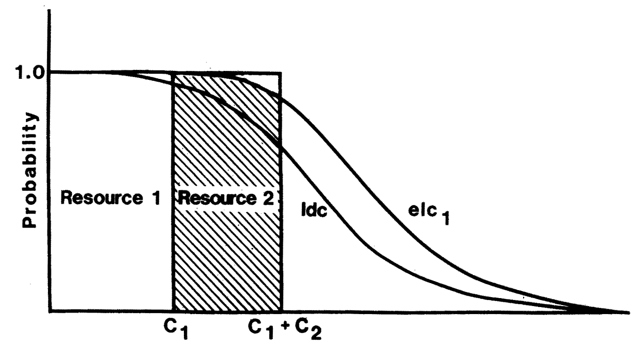


Fig 2. Dispatching the Second Resource

Finding the energy served by the second resource is a process similar to that used for the first resource. However, the second resource must be dispatched against the load which it faces: the equivalent load curve after dispatch of the first unit. The shaded area under the equivalent load curve in Figure 2 is multiplied by the probability of the resource serving load to obtain the energy served by the resource. The probability density of the equivalent load after the second resource is dispatched (the load that faces the third resource) is found by convolving the outages of the second unit with the equivalent load curve that the second unit faced

$$el_2(EL_2) = \int_D el_1(D)\, o_2(EL_2 - D)$$

This process is repeated until all the resources have been dispatched, determining the energy served by each resource.

An alternative method of calculating the energy served by a resource [9] is often used in practice and is used when developing the derivatives in Section IV. The total energy per unit of time associated with a load duration curve is the integral of the load duration curve. After m resources are dispatched, the remaining unserved energy per unit of time is the integral of the remaining load plus outages of the first m units. Thus, the integral of interest is of the equivalent load curve over the region that exceeds the capacity dispatched in the first m resources. Let $NUE_m$ be the unserved energy per unit of time after m resources are dispatched. (The N indicates that the unserved energy is normalized per unit of time.) These m resources contain $CD_m$ megawatts of capacity, where

$$CD_m = \sum_{i=1}^{m} C_i$$

The unserved energy per unit of time after m resources are dispatched is

$$NUE_m = \int_{EL_m > CD_m} elc_m(EL_m)$$

The energy served by resource m per unit of time is simply the unserved energy before the resource is dispatched less the unserved energy after the resource is dispatched. That is

$$NES_m = NUE_{m-1} - NUE_m \tag{1}$$

## III. REPRESENTING THE EQUIVALENT LOAD CURVE

The production simulation method described in Section II requires the integral of the equivalent load curve. To calculate this integral an algebraic or numerical representation of the equivalent load curve is needed. Several methods are used often in practice. First, the equivalent load curve can be represented numerically as a table (see, for example, [8]). The convolution of the outages of each resource is performed numerically. Second, the curve can be represented using a Fourier Series [10]. Recently, a third, more computationally efficient method has been developed [11]. We use the third method, called the cumulant method, in this paper.

The cumulant method represents the probability density function f(X) as a series expansion of the normal distribution (n(X)) and its derivatives ($n^i(X)$)

$$f(X) = A_0\, n(X) + A_1\, n^1(X) + \frac{A_2}{2!}\, n^2(X) + \ldots \tag{2}$$

To determine the $\{A_i\}$ we use the Hermite polynomials. These are a series of polynomials ($h_i(X)$), one of each order, with several desirable properties [12].

We proceed by using an identity from [12]

$$n^i(X) = (-1)^i\, h_i(X)\, n(X)$$

By substituting in (2) we obtain

$$f(X) = \sum_{i=0}^{\infty} A_i\, h_i(X)\, n(X)\, \frac{(-1)^i}{i!} \tag{3}$$

Next, we use a second indentity from [12]

$$\int_X h_i(X)\, h_j(X)\, n(X) = \begin{cases} i! & i = j \\ 0 & \text{otherwise} \end{cases}$$

Each coefficient can be found by multiplying (3) by $h_i(x)$ and integrating

$$\int_X h_i(X)\, f(X) = (-1)^i A_i$$

or

$$A_i = (-1)^i \langle h_i(X) \rangle \tag{4}$$

where the brackets denote expectation.

The expected values of the Hermite polynomials are linked to the probability density in an important manner. Cumulants, which we denote by $k_i$, are functions of the central moments ($M_i$) of f(X) [12]. The expected value of each Hermite polynomial is a function of the cumulants [12]. Thus, calculating the $\{A_i\}$ in the series shown in (2) is simple when the moments or cumulants of f(X) are known, relieving us of the tedium of the complex integration shown in (4).

The production simulation uses the cumulants to represent the coefficients of the infinite series in (2) rather than the central moments because of an important property of cumulants: the ith cumulant of the convolution of a set of independent random variables is the sum of the ith cumulant of each random variable in the set. Let X and the set $\{Y_i\}$ be random variables, with

$$X = \sum_i Y_i$$

Let f(X) and $g_i(Y_i)$ be the probability density functions of the random variables. The jth cumulant of the probability density function f(X) is $k_j(f(X))$. Then

$$k_j(f(X)) = \sum_i k_j(g_i(Y_i))$$

if the $\{Y_i\}$ are independent.

## IV. METHOD FOR CALCULATING THE DERIVATIVES

Numerous derivatives of production simulation results could help guide power planning. However, they all possess a characteristic that allows development of a method for obtaining these derivatives. All of the derivatives are with respect to variables whose change can be related to changes in the cumulants. For example, the change in the energy served by a resource with respect to the capacity of a resource dispatched prior to it can be traced through the cumulants.

The derivative we desire is $\partial ES_j / \partial C_i$, the change in energy served by resource j with respect to a change in the capacity of the ith resource. Equation (1) provided a method for determining the normalized energy served by resource i. The corresponding equation for the actual energy served is

$$ES_j = UE_{j-1} - UE_j \tag{5}$$

where $UE_i$ is the actual unserved energy after dispatching the ith resource. Differentiating (5) with respect to $C_i$ yields

$$\frac{\partial ES_j}{\partial C_i} = \frac{\partial UE_{j-1}}{\partial C_i} - \frac{\partial UE_j}{\partial C_i} \tag{6}$$

Thus the problem is reduced to that of finding $\partial UE_j / \partial C_i$. Calculating any other derivative of interest uses the same approach. The variable of interest replaces the capacity $C_i$.

The first step in finding this derivative is finding the derivative of normalized (per unit of time) and standardized (mean zero, variance one) unserved energy, NSUE, with respect to the cumulants. Normalized and standardized unserved energy is given by the following equation from [9].

$$NSUE_j = \int_{Z_j \geq CD_j} selc_j(Z_j) \tag{7}$$

$$= \int_{Z_j}^{\infty} \int_{Y}^{\infty} N(W)\, dW dY - \frac{g_{3j}}{6}\, n^1(Z_j) + \ldots \quad (Y = Z_j)$$

where

$$CD_j = \sum_{1}^{j} C_i$$

$$Z_j = (CD_j - M_j)/\sigma_j$$

$M_j$ = mean = 1st cumulant = $k_{1j}$

$\sigma_j^2$ = variance = 2nd cumulant = $k_{2j}$

$g_{sj}$ = standardized cumulant = $k_{sj}/\sigma_j^s$

$$k_{sj}(elc_j) = \sum_{i=1}^{j} k_s^i + k_s(d(D))$$

$k_s^i$ = $k_s(o_i(O_i))$ = sth cumulant for the ith resource's outage

$k_s(d(D))$ = sth cumulant of the load

$n^m(Z)$ = mth derivative of the normal distribution at Z

$selc_j(Z_j)$ = standardized equivalent load curve

Applying integration by parts to (7) and including the terms that use the first six cumulants [11], we have

$$\begin{aligned} NSUE_j = {} & -Z_j \int_{Z_j} n(W)\, dW + n(Z_j) - \frac{g_{3j}}{6} n^1(Z_j) \qquad (8) \\ & + \frac{g_{4j}}{24} n^2(Z_j) \\ & + \frac{g_{3j}^2}{72} n^4(Z_j) \\ & - \frac{g_{5j}}{120} n^3(Z_j) \\ & - \frac{g_{3j}g_{4j}}{144} n^5(Z_j) \\ & - \frac{g_{3j}^3}{1296} n^7(Z_j) \\ & + \frac{g_{6j}}{720} n^4(Z_j) \\ & + \frac{g_{4j}^2}{1152} n^6(Z_j) \\ & + \frac{g_{3j}g_{5j}}{720} n^6(Z_j) \\ & + \frac{g_{3j}^2 g_{4j}}{1728} n^8(Z_j) \\ & + \frac{g_{3j}^4}{31104} n^{10}(Z_j) \end{aligned}$$

Now let us calculate $\partial NSUE_j/\partial k_s$. We will do the calculations only for the first three terms on the right-hand side of (8); the calculations for the other terms are similar.

$$\begin{aligned} \frac{\partial NSUE_j}{\partial k_{sj}} = {} & - \frac{\partial Z_j}{\partial k_{sj}} \int_{Z_j}^{\infty} n(W)\, dW - Z_j \frac{\partial}{\partial k_{sj}} \int_{Z_j}^{\infty} n(W)\, dW \\ & + \frac{\partial n(Z_j)}{\partial k_{sj}} - \frac{\partial g_{3j}}{\partial k_{sj}} \frac{n^1(Z_j)}{6} - \frac{g_{3j}}{6} \frac{\partial n^1(Z_j)}{\partial k_{sj}} \\ = {} & - \frac{\partial Z_j}{\partial k_{sj}} \int_{Z_j}^{\infty} n(W)\, dW + Z_j \frac{\partial Z_j}{\partial k_{sj}} n(Z_j) \\ & + \frac{\partial n(Z_j)}{\partial Z_j} \frac{\partial Z_j}{\partial k_{sj}} - \frac{\partial g_{3j}}{\partial k_{sj}} \frac{n^1(Z_j)}{6} \\ & - \frac{g_{3j}}{6} \frac{\partial n^1(Z_j)}{\partial Z_j} \frac{\partial Z_j}{\partial k_{sj}} \end{aligned}$$

Using the fact that $\partial n(Z)/\partial Z = -Z\, n(Z)$, the second and third terms cancel each other out so that we have

$$\begin{aligned} \frac{\partial NSUE_j}{\partial k_{sj}} = {} & - \frac{\partial Z_j}{\partial k_{sj}} \int_{Z_j}^{\infty} n(W)\, dW - \frac{\partial g_{3j}}{\partial k_{sj}} \frac{n^1(Z_j)}{6} \\ & - \frac{\partial Z_j}{\partial k_{sj}} \frac{g_{3j} n^2(Z_j)}{6} \end{aligned} \qquad (9)$$

Next, let us calculate $\partial Z_j/\partial k_{sj}$. Since

$$Z_j = \frac{CD_j - M_j}{\sigma_j} = \frac{CD_j - k_{1j}}{k_{2j}^{1/2}}$$

we obtain:

$$\frac{\partial Z_j}{\partial k_{sj}} = \begin{cases} -k_{2j}^{-1/2} & s = 1 \\ -\dfrac{Z_j}{2k_{2j}} & s = 2 \\ 0 & s \geq 3 \end{cases} \qquad (10)$$

We also need $\partial g_{qj}/\partial k_{sj}$ for values of q greater than 2. Since $g_{qj} = \sigma_j^{-q} k_{qj}$, we have

$$\frac{\partial g_{qj}}{\partial k_{sj}} = \begin{cases} \sigma_j^{-s} & s = q \\ -\dfrac{q}{2k_{2j}} g_{qj} & s = 2 \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

Substituting (10) and (11) into (9), we obtain the derivatives of standardized and normalized unserved energy with respect to the first three cumulants:

$$\frac{\partial NSUE_j}{\partial k_{1j}} = \frac{1}{\sigma_j}\left[\int_{Z_j}^{\infty} n(W)\, dW + \frac{g_{3j}}{6}\, n^2(Z_j)\right]$$

$$\frac{\partial NSUE_j}{\partial k_{2j}} = \frac{Z_j}{2\sigma_j^2}\left[\int_{Z_j}^{\infty} n(W)\, dW + \frac{g_{3j}}{6}\, n^2(Z_j)\right] + \left(\frac{n^1(Z_j)}{6}\right)\left(\frac{3}{2\sigma_j^2}\right) g_{3j}$$

$$\frac{\partial NSUE_j}{\partial k_{3j}} = -\frac{n^1(Z_j)}{6}\,\frac{1}{\sigma_j^3} \qquad (12)$$

The derivatives with respect to the higher order cumulants are calculated in a similar manner.

Now we are ready to calculate the derivatives of actual unserved energy with respect to capacity.

$$UE_j = H\,\sigma_j\, NSUE_j \qquad (13)$$

$$= H\, k_{2j}^{1/2} NSUE_j(Z,\, k_{1j},\, k_{2j},\, k_{3j},\, k_{4j},\, k_{5j},\, k_{6j})$$

where H is the number of hours in the period over which the production simulation is being carried out. Differentiating (13) with respect to capacity we obtain

$$\frac{\partial UE_j}{\partial C_i} = H\left\{k_{2j}^{1/2}\,\frac{\partial NSUE_j}{\partial C_i} + NSUE_j\,\frac{\partial k_{2j}^{1/2}}{\partial C_i}\right\}$$

$$= H\left\{k_{2j}^{1/2}\sum_{s=1}^{6}\left[\left(\frac{\partial NSUE_j}{\partial k_{sj}}\right)_{z_j,k_{nj},n\neq s}\frac{\partial k_{sj}}{\partial C_i}\right] + k_{2j}^{1/2}\left(\frac{\partial NSUE_j}{\partial Z_j}\right)_{k_n;\ \text{all } n} \times \frac{\partial Z_j}{\partial C_i} + NSUE_j\,\frac{\partial k_{2j}^{1/2}}{\partial C_i}\right\} \qquad (14)$$

Several of the partial derivatives on the right-hand side of (14) remain to be calculated. The resource outage cumulants, $k_s^i$ are for each resource as a whole, but a resource may consist of multiple identical units. Let $k_1^i$ and $k_2^i$ be the first and second outage cumulants, respectively, of one unit of resource i. Then

$$\frac{\partial k_{sj}}{\partial C_i} = uk_s^i \qquad (15)$$

where

$$uk_s^i = \frac{k_s^i}{S_i}$$

$k_s^i$ = outage cumulant for nominal unit of resource i

$S_i$ = size of nominal unit of resource i

Next recall that

$$Z_j = \frac{CD_j - M_j}{\sigma_j} = \frac{CD_j - k_{1j}}{k_{2j}^{1/2}}$$

Differentiating, we obtain

$$\frac{\partial Z_j}{\partial C_i} = \frac{1}{\sigma_j}(1 - uk_1^i) - (CD_j - k_{1j})\,\frac{uk_2^i}{2\sigma_j^3}$$

$$= \frac{1}{\sigma_j}(1 - uk_1^i) - Z_j\,\frac{uk_2^i}{2\sigma_j^2} \qquad (16)$$

Note that (15) and (16) are valid only for $j \geq i$. The right-hand sides of these equations are zero for $j < i$, as is that of (14) by substitution; the output of resource j is unaffected by any parameters of resource i when resource j is dispatched prior to resource i. The remaining partial derivative is found by differentiating (8) with respect to $Z_j$.

$$\left(\frac{\partial NSUE_j}{\partial Z_j}\right)_{k_{nj};\ \text{all } n} = -\int_{Z_j}^{\infty} n(W)\, dW \qquad (17)$$

$$-\frac{g_{3j}}{6}\, n^2(Z_j) + \frac{g_{4j}}{24}\, n^3(Z_j)$$

$$+\frac{g_{3j}^2}{72}\, n^5(Z_j) - \frac{g_{5j}}{120}\, n^4(Z_j)$$

$$-\frac{g_{3j}g_{4j}}{144}\, n^6(Z_j) - \frac{g_{3j}^3}{1296}\, n^8(Z_j)$$

$$+\frac{g_{6j}}{720}\, n^5(Z_j) + \frac{g_{4j}^2}{1152}\, n^7(Z_j)$$

$$+\frac{g_{3j}g_{5j}}{720}\, n^7(Z_j) + \frac{g_{3j}^2 g_{4j}}{1728}\, n^9(Z_j)$$

Substituting (8), (12), (15), (16), and (17) into (14) yields $\partial UE_j/\partial C_i$, which is then substituted into (6) to provide the desired derivative. While these algebraic manipulations would be time-consuming if done by hand, they are extremely fast when done by computer, particularly since most of them have to be done anyway as part of the production simulation.

## V. VERIFICATION

The derivative calculations have been implemented in the production simulation contained in the Strategic Analysis Model used by TVA [5]. This production simulation uses the change in unserved energy to evaluate the energy served by a resource and represents the equivalent load duration curve with the first six terms of the infinite series of the normal distribution and its derivatives. The coefficients of the series are calculated using the cumulants of the probability distributions.

A synthetic utility system used to test new production simulations here and elsewhere [13] is called Scaled Down EPRI System D. Table I describes the resources for the system. The system load is normally distributed with mean 6267.4 megawatts and standard deviation 745.4 megawatts. The period for which resources are being dispatched is 728 hours.

Table I

SCALED DOWN EPRI D SYSTEM RESOURCES

| Loading Order | Unit Size (MW) | Number of Units | Total MW | Forced Outage Rate |
|---|---|---|---|---|
| 1 | 1200 | 2 | 2400 | 0.15 |
| 2 | 800 | 1 | 800 | 0.24 |
| 3 | 600 | 2 | 1200 | 0.21 |
| 4 | 400 | 2 | 800 | 0.13 |
| 5 | 400 | 1 | 400 | 0.13 |
| 6 | 200 | 8 | 1600 | 0.074 |
| 7 | 200 | 7 | 1400 | 0.074 |
| 8 | 50 | 29 | 1450 | 0.24 |

The calculations using the formulas developed in Section IV were compared to those obtained by numerical methods when a one megawatt addition was made to a particular resource. Tables II and III present the results. The results of using the two methods are very similar. The magnitude of the errors is small, and except for the case where the change is small in magnitude (resource 5 in Table III), the differences are all less than one percent. Since each calculation uses approximation techniques to solve a very complex probabilistic mapping, neither can be viewed as a bench mark.

Table II

DERIVATIVE TEST 1

| Position in Loading Order | $\frac{\partial E_J}{\partial C_5}$ (MWh/MW) | Change in $E_j$ When One MW of Resource 5 is Added |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 0 | 0 |
| 3 | 0 | 0 |
| 4 | 0 | 0 |
| 5 | +594 | +592 |
| 6 | -264 | -263 |
| 7 | -236 | -235 |
| 8 | -75 | -75 |
| Energy Not Served | -19 | -19 |

Table III

DERIVATIVE TEST 2

| Position in Loading Order | $\frac{\partial E_J}{\partial C_4}$ (MWh/MW) | Change in $E_j$ When One MW of Resource 4 is Added |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 0 | 0 |
| 3 | 0 | 0 |
| 4 | +617 | +616 |
| 5 | -22 | -24 |
| 6 | -264 | -263 |
| 7 | -236 | -235 |
| 8 | -75 | -75 |
| Energy Not Served | -20 | -19 |

## VI. APPLICATION

This section shows two ways in which the derivatives are used in the Strategic Analysis Model [5]. The first is in capacity expansion planning. Suppose that resource 4, in the example of Section V, consists of coal-fired plants with a variable cost of 20 mills/kWh; resource 5 is an oil-fired plant with variable cost of 50 mills/kWh; and resources 6, 7, and 8 have variable costs of 55 mills/kWh, 60 mills/kWh, and 70 mills/kWh, respectively. Table IV shows how multiplying the derivatives in Table II by these variable costs and summing yields the change in system operating cost that would result from having one additional megawatt of resource 5. The reduction in fuel cost for the 728-hour period (one month) is $4230 if one megawatt of resource 5 is added. When calculating the benefits of adding additional capacity, the increase in system reliability as well as the reduction in operating cost should be considered. A frequently used method for measuring the increased reliability is assigning an outage cost to expected energy not served, that part of load that remains unsatisfied even after all emergency actions have been taken. Suppose that for system planning purposes the utility in this example assigns a value of $1.50/kWh to energy not served. Then adding the value of reducing energy not served to the operating savings yields a reduction in social cost (operating cost plus outage cost) of $32 730 for adding one megawatt of resource 5. The corresponding cost reductions for adding one megawatt of resource 4 (coal) are $22 690 in fuel cost alone and $51 190 when reduction in outage cost is included.

Table IV

CHANGE IN OPERATING COST

| Position in Loading Order | $\frac{\partial E_J}{\partial C_5}$ (MWh/MW) | Variable Cost ($/MWh) | Change in System Operating Cost ($) |
|---|---|---|---|
| 1 | 0 | 10 | 0 |
| 2 | 0 | 15 | 0 |
| 3 | 0 | 18 | 0 |
| 4 | 0 | 20 | 0 |
| 5 | +594 | 50 | 29 700 |
| 6 | -264 | 55 | -14 520 |
| 7 | -236 | 60 | -14 160 |
| 8 | -75 | 70 | -5 250 |
| Energy Not Served | -19 | 1500 | -28 500 |
| | | Total | -32 730* |

*-4 230 fuel; -28 500 outage.

To use these results in capacity expansion planning, we perform a similar calculation for each period that the incremental megawatt will remain in service and take the discounted present value of the resulting savings for each of the resources for which the utility might add additional capacity. If the present value of the operating savings is greater than the present value of fixed owning and operating costs for a resource, then the total system costs will be reduced by adding capacity of that resource.

Another application of the derivatives in the Strategic Analysis Model [5] is for maintenance scheduling. Suppose that the objective in scheduling is to minimize the total expected energy not served throughout the year. The derivative of energy not served with respect to capacity (the bottom row in Tables II, III, and IV) can be compared across periods. By moving maintenance from a period in which the absolute value of this derivative is low to one in which the absolute value is high, the total expected energy not served is reduced.

## VII. CONCLUSIONS

In Section IV, we calculated the derivative of energy served by a resource with respect to the capacity of any other resource. From there, it is straightforward to calculate the derivative of system operating cost with respect to capacity of any resource. Combining this with the change in fixed costs resulting from adding capacity produces derivatives of total system cost, which can be used in determining an optimal capacity expansion plan. Similar derivatives can be used for maintenance scheduling and for calculating marginal costs with respect to load.

The cumulant implementation of the equivalent load methodology is an extremely fast production simulation algorithm. Calculating the derivatives using the analytical approach derived in Section IV requires only about ten percent more computation time than running one production simulation. Calculating the derivatives numerically, on the other hand, would require running the production simulation n times, once for each resource, increasing computation time by a factor of n. The ease and speed with which the derivatives can be calculated using the analytical approach commends the approach as a powerful tool for electric power system planning.

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## APPENDIX A: NOTATION

We use the integral sign to indicate integration or summation. Either the limits of integration are represented as a set below the integral sign or the limits are explicitly shown above and below the integral sign. Thus, for example, if S is the real line,

$$\int_S f(X) = \int_{-\infty}^{\infty} f(X)\, dX$$

We use the term probability distribution function to denote one of the cumulative probability functions of a probability density or mass function. Let f(X) be a density function, then,

$$\int_{Y \leq X} f(Y)$$

is the cumulative probability distribution function, while

$$1 - \int_{Y \leq X} f(Y)$$

is the complementary cumulative distribution function.

The remaining notation pertains to functions and variables. We represent functions using lower case letters. Thus, both probability density functions and distribution functions appear in lower case letters. Every probability distribution function used in this paper has a historically important name, so our convention will not confuse the density and distribution functions. We represent variables and random variables using upper case letters. We call some variables normalized, meaning that they represent a value per unit of time. We call other variables standardized, meaning that they have mean zero and variance one.

The specific notation is as follows:

$C_i$ = capacity of the ith resource

$CD_i$ = the total amount of capacity dispatched after the ith resource has been dispatched

D = demand

d = probability density on demand

$EL_i$ = the equivalent load after i resources have been dispatched

$el_i$ = the probability density on the equivalent load after i resources have been dispatched

$elc_i$ = the equivalent load curve after i resources have been dispatched, a complementary probability distribution

$ES_i$ = the energy served by the ith resource during the period

$g_i(f)$ = the standardized ith cumulant of probability density function f

H = number of hours in the period

$h_i(X)$ = the Hermite polynomial of order i

$k_i(f)$ = the ith cumulant of the probability density function f

$k_{ij}(elc_j)$ = the ith cumulant of the equivalent load curve after j resources have been dispatched

$k_s^i(o_i)$ = the sth cumulant of a unit of the ith resource

ldc = load duration curve, the complementary cumulative distribution on demand D

n(X) = the normal probability density function on X

$n^i(X)$ = the ith derivative of the normal density function on X

$NES_i$ = the energy served by the ith resource per unit of time

$NSUE_i$ = the standardized unserved energy per unit of time after i resources have been dispatched

$NUE_i$ = the unserved energy per unit of time after i resources have been dispatched

$O_i$ = the outage capacity of the ith resource

$o_i$ = probability density on the outage capacity of the ith resource

$selc_i$ = the standardized equivalent load curve after i resources have been dispatched, a complementary probability distribution

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## Discussion

**K. F. Schenk** (International Atomic Energy Agency, Vienna, Austria): This is an interesting and important paper. The authors exploit the cumulant method to evaluate the derivatives of energy served by a resource with respect to the capacity of any other resource. The numerical evaluations of these derivatives would be computationally too expensive to use in practice. The analytical evaluation of these derivatives is very efficiently performed by the cumulant method, as this paper attests.

However, it must be observed that the remarkable agreement that the authors have obtained in their results may be due in part to the fact that their system has a load that is normally distributed and a generation system that has small values for the standardized cumulants.

The discusser (1) has analyzed a similar system as the one used by the authors and the results obtained by using the Gram-Charlier expansion and the cumulant method are quite acceptable. Levy and Kahn (2) have also provided some guidelines on the accuracy of the Edgeworth expansion for LOLP calculations. It is well known that the cumulant method may experience difficulties for small systems with disparate unit sizes and low FORs.

In the interest of completeness, would be authors consider rerunning their algorithm but using a modified system? In fact, I suggest the following: (i) use the same load distribution but divide the FORs of all units by a factor of 3, (ii) use the same generation system but a different load distribution, preferably bimodal, and (iii) combine (i) and (ii).

In closing, I would like to congratulate the authors for a well-written paper.

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**J. Bloom,** (GPU Service Corp., Parsippany, NJ): I would like to call the attention of the authors to the following previously published papers on this subject:

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